

NO-A143 641

RESEARCH ON TOPICS IN PERTURBATION METHODS TRANSONIC
FLOW THEORY NUMERICAL (U) NIELSEN ENGINEERING AND
RESEARCH INC MOUNTAIN VIEW CA D NIXON ET AL. APR 84

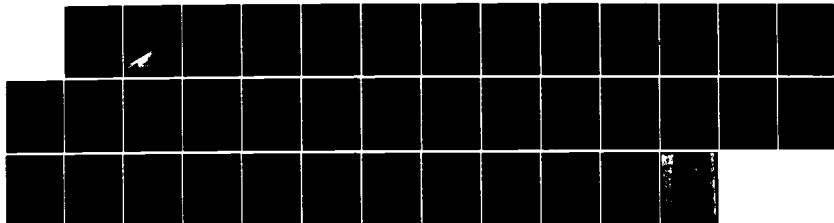
1/1

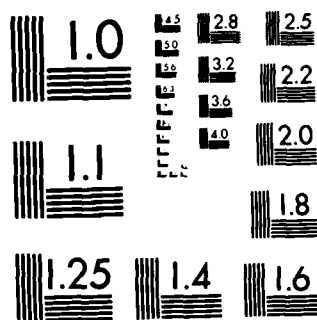
UNCLASSIFIED

NEAR-TR-324 AFOSR-TR-84-0540

F/G 5/1

NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

AD-A143 641

RESEARCH ON TOPICS IN PERTURBATION METHODS,
TRANSONIC FLOW THEORY, NUMERICAL ANALYSIS,
AND ADAPTIVE GRID GENERATION

by

David Nixon and Goetz H. Klopfer

This report covers the period
April 1, 1979 to January 31, 1984

DTIC FILE COPY

Approved for public release;
Distribution unlimited.

**NIELSEN ENGINEERING
AND RESEARCH, INC.**

OFFICES: 510 CLYDE AVENUE / MOUNTAIN VIEW, CALIFORNIA 94043 / TELEPHONE (415) 968-9457

84 07 24 027

RESEARCH ON TOPICS IN PERTURBATION METHODS,
TRANSONIC FLOW THEORY, NUMERICAL ANALYSIS,
AND ADAPTIVE GRID GENERATION

by

David Nixon and Goetz H. Klopfer

This report covers the period
April 1, 1979 to January 31, 1984

NEAR-TR-324 ✓

April 1984

Prepared Under Contract No. F49620-79-C-0054

for

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
Bolling Air Force Base
Washington, DC 20332

by

NIELSEN ENGINEERING & RESEARCH, IN.
510 Clyde Avenue, Mountain View, CA 94043
Telephone (415) 968-9457

MAILED
Chief, Information Division

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM	
1. REPORT NUMBER AFOSR-TR. 84-0518		2. GOVT ACCESSION NO. AD-A143641	
3. TITLE (and Subtitle) RESEARCH ON TOPICS IN PERTURBATION METHODS, TRANSONIC FLOW THEORY, NUMERICAL ANALYSIS AND ADAPTIVE GRID GENERATION		5. TYPE OF REPORT & PERIOD COVERED Final 4/1/79 - 1/31/84	
7. AUTHOR(s) D. Nixon and G. H. Klopfer		6. PERFORMING ORG. REPORT NUMBER TR 324	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Nielsen Engineering & Research, Inc. 510 Clyde Avenue Mountain View, CA 94043		8. CONTRACT OR GRANT NUMBER(s) F49620-79-C-0054	
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NA Bolling Air Force Base Washington, DC 20332		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2307 / A1	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE April 1984	
		13. NUMBER OF PAGES 39	
		15. SECURITY CLASS. (of this report) Unclassified	
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE	
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; Distribution unlimited.			
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)			
18. SUPPLEMENTARY NOTES			
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Perturbation Methods Transonic Flow Numerical Analysis Adaptive Grids			
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report summarizes the work performed during the period April 1, 1979 to January 31, 1984 under the sponsorship of AFOSR Contract F49620-79-C-0054. The work is concerned with certain topics on perturbation theory, transonic flow theory, numerical analysis, and adaptive grid generation. The perturbation theory for transonic flow was further developed for two- and three-dimensional problems. The physical			

UNCLASSIFIED

Block 20. (Abstract) continued.

perturbation theory, in which the perturbation parameter is a physical quantity, is applied to solutions of the Navier-Stokes equations in two dimensions. The mathematical perturbation theory, in which the perturbation parameter is a measure of the difference between approximate and exact solutions, is applied both to the three-dimensional potential flow problems and to the two-dimensional Navier-Stokes equations. The strained coordinate technique is used to treat changes in locations of any shock waves or large gradients. In addition to these steady applications of the perturbation theory, both the physical and mathematical versions of the technique have been satisfactorily extended to two-dimensional unsteady flow.

An indicial response function for transonic flow that does not require a lengthy transonic computation was also developed. This investigation produced a means of radically reducing the computation time needed for an aeroelastic calculation. A theory has been developed which requires steady state results at either extremity of the indicial response and some estimate of the initial time derivatives.

In transonic flow theory the extension of the transonic perturbation method to include flows where shock waves vanish and the development of the technique to treat separated flows was undertaken with satisfactory results. Two other topics that were investigated concerned the application of perturbation theory to accelerate convergence of numerical solutions to predict potential flow. Finally, for transonic flow, the development of a "potential-like" theory to more closely approximate the Euler equations was undertaken.

Some other work on transonic flow theory was concerned with the existence of multiple solutions in full potential calculations. Since the full potential equation is difficult to analyze compared with the small disturbance equation, multiple solutions have been found using transonic small disturbance theory. These results have been analyzed using the transonic integral equation theory and indicate that the transonic potential theory is not formulated uniquely.

A non-linear truncation error analysis was performed on certain Euler equation algorithms to develop corrections for the solution. An outcome of this work was the derivation of a criteria for use in adaptive mesh techniques. The work on adaptive mesh procedures was concerned with the development of adaptive mesh strategies and solution procedures for highly clustered adaptive meshes. It has been found that the strong conservation law form of the governing equations in computational variables cannot capture the shock waves correctly for arbitrary clustering. Methods for correcting this problem have been investigated.

A rational basis for generating solution adaptive meshes for the unsteady two-dimensional Euler equations was developed for both explicit and implicit numerical algorithms. It was found that for unsteady flows, the mesh generation consumed most of the computational effort required which can negate the efficiency gained by the reduced number of grid points possible with clustered grids.

TABLE OF CONTENTS

	Page
INTRODUCTION	1
FINAL STATUS OF THE RESEARCH EFFORT	4
Perturbation Theory for Transonic Flow	4
Development of a Universal Indicial Function	6
Perturbation Theory with Vanishing Shock Waves	7
Convergence Acceleration	8
Strong Shock Potential Theory	8
Convergence Acceleration of Euler Equations	9
Truncation Error Analysis	9
Adaptive Meshes	9
Transonic Multiple Solutions	13
PERSONNEL	14
PAPERS AND REPORTS	14
PRESENTATIONS	16
TECHNICAL APPLICATIONS	17
REFERENCES	18
FIGURES	21

Accession For

CLASSI

215



A1

RESEARCH ON TOPICS IN PERTURBATION METHODS,
TRANSONIC FLOW THEORY, NUMERICAL ANALYSIS,
AND ADAPTIVE GRID GENERATION

INTRODUCTION

The investigations conducted under AFOSR contract F69620-79-C-0054 constitute a variety of subjects concerned with transonic flow theory and numerical analysis. The topics concerned with transonic flow are perturbation methods and the validity of potential theory. The numerical analysis study is concerned with the effects of truncation error on the solution.

The first subject to be studied is a theoretical investigation for the development and examination of validity of predictive techniques for determining solutions which represent perturbations in both physical (geometry, flow conditions) and nonphysical (grid refinement, level of approximation) parameters for nonlinear transonic flows past aerodynamic configurations. The primary goal is to develop procedures to extend and enhance the utility of currently emerging advanced finite-difference computer codes for calculating both two- and three-dimensional, steady and unsteady, inviscid and viscous transonic flows by substantially reducing the computational requirements of these codes when applied to large number of related cases, as in parametric analysis for design or operational studies. In addition, approximate theories based on these perturbation ideas are developed which have an application in transonic flutter analysis.

The second topic is concerned with the development of a universal transonic indicial function in order to avoid lengthy computations in deriving such functions. This study was only partially successful.

The perturbation theories studied during the initial part of the contract are restricted to flows in which shock waves are not generated or destroyed during the perturbation. This can be a fairly severe restriction and therefore a study to remove this restriction was initiated with satisfactory results.

A consequence of perturbation theory is that it is theoretically possible to devise means of accelerating convergence. Thus, a study to investigate the possibility of accelerating convergence of potential equation solutions by using the perturbation theory was initiated. Two approaches were studied, namely:

- a. a grid refinement technique using a sequence of three or more grids and
- b. use the strained coordinate method in conjunction with the classic ideas of convergence acceleration.

In addition to the above topics the following investigations, connected with transonic flow theory, but not necessarily with the transonic perturbation theory, were conducted:

Examine the possibility of using a single equation to accelerate the convergence of the Euler or Navier-Stokes solutions.

Examine the possibility of modeling strong shocks in a potential formulation by using the concept of an internal energy to represent the effect of the vorticity transport.

Further work on transonic flow theory concerns an investigation into the occurrence of multiple solutions for a class of flow parameters. Specifically, the occurrence of lifting solutions for symmetric airfoils at zero angle of attack.

The other topics, not related to transonic flows, investigated in this contract are concerned with numerical analysis of finite difference schemes.

The research objectives in numerical analysis are about the derivation of the nonlinear truncation errors for the generalized Lax-Wendroff schemes and the implicit scheme of Beam and Warming for the two-dimensional Euler equations. Secondly, the implementation of the correction procedure to suitable existing practical codes based on the MacCormack scheme and the implicit scheme. An outcome of this basic idea is the extension to adaptive mesh generation techniques so that accurate finite difference solutions to a set of nonlinear partial differential equations can be obtained with the minimum number of mesh points. Some preliminary work on adaptive mesh procedures based on nonlinear truncation error analysis indicated four basic problems that need to be resolved: (1) clustering makes the numerical solution of the transformed equations more difficult due to the extra stiffness introduced in the partial differential equations; (2) proper clustering functions are necessary to minimize the truncation errors; (3) the truncation errors must be filtered and smoothed before they are suitable for use as clustering criteria; and (4) artificial dissipation (probably depending on the local mesh size) must be introduced to guarantee smooth and monotonic solutions at shock waves and other flow discontinuities.

The final piece of work on this contract involved the generation of solution adaptive meshes for the two-dimensional unsteady Euler equations solved by a second order accurate explicit and implicit scheme. This required the extension of the nonlinear truncation error analysis to the unsteady Euler equations on an arbitrarily moving curvilinear coordinate system. The analysis provided for a rational basis for clustering the mesh points. To obtain an efficient grid generator

several mesh generators were to be investigated and a weighted interpolation scheme devised so that an initially very coarse mesh could be used as the starting point for clustered mesh.

Final Status of the Research Effort

Perturbation Theory for Transonic Flow. The main object is to extend some developments in perturbation theories of transonic flow. One topic is the extension of the physical perturbation theory to treat solutions of the Navier-Stokes equation. A second topic is the investigation into the possibility of correcting lower grade inviscid solutions for viscous effects using the mathematical perturbation theory. Thirdly, the extension of the mathematical perturbation theory for potential flows to three dimensions is considered. The extension of the physical perturbation theory (references 1 and 2) to solutions of the Navier-Stokes equations is straightforward, the only additional fact to appear is that the necessary base and calibration solutions should not be too close together, otherwise the perturbation quantities can be seriously degraded by the numerical error in the solution. A typical solution is shown in figure 1; a result for separated flow is given in figure 2.

Application of the correction technique to upgrade potential equation solutions to include viscous effects introduced some interesting problems. The main difficulty proved to be due to the fact that, for relatively strong shock waves, the potential theories give shock locations that are much further aft than those predicted by the Euler or the Navier-Stokes equations. This raised problems in the application of the concept of nearby solutions, since the sensitivity of some of the examples to small geometry changes could allow a flow change from one with a relatively weak shock to one with a strong shock. This led to the

necessity for a correction (reference 3) for the purely inviscid effects due the use of the potential equation rather than the Euler equations. Examples such as that shown in Figure 3, compared with the subsequent correction theory compare adequately with direct calculations.

The extension of the mathematical correction theory to three-dimensions is a straightforward development of the two-dimensional theory. Both corrections for grid size and for the use of the transonic small disturbance (TSD) equation are considered. Within certain limits the perturbation method appears satisfactory. A result for the ONERA M6 wing is shown in figure 4. However, in all of the topics discussed above, it is desirable that a more comprehensive testing procedure be performed over a wider range of airfoils and wings in order to assess the range of applicability of the theories.

The direct unsteady flow analogy of the steady perturbation theory is the transonic indicial theory developed in reference 4. In the present work the formulae developed in reference 4 have been incorporated in a computer program CONVOL and the results are very satisfactory. A typical result is shown in figure 5 and compared to the results of the Ballhaus-Goorjian code LTRAN2.

Using CONVOL can result in a very substantial savings in computation time, particularly if a range of frequencies is to be run. In fact, the pressure distributions for about ten frequencies can be computed in the time that LTRAN2 requires for one. In the case of multi-parameter perturbations, the potential for savings is even greater, since an entire n -parameters space of solutions can be run for little more than the time required for $(n+1)$ finite difference calculations. The indicial method is also immediately generalizable to three dimesions.

The multi-parameter capability, together with the reduced computation times, opens up the possibility for using CONVOL to fit the aerodynamic response to the structural model in a combined program. This "tailoring" capability ought to be of particular value in the development of active control technology for aircraft.

The extension of the mathematical perturbations theory (reference 5) to unsteady flow is a straightforward development of the theory described in reference 5. The correction at each time step is obtained by computing one exact and one approximate solution and taking the difference. The strained coordinate method is used to treat shock waves and any rapid gradients in the flow. The only difference between steady and unsteady applications of the theory is that in unsteady applications the correction is applied at each time step. Hence, both coarse and fine grid solutions for all cases must be at the same time step.

In figure 6 the corrected coarse grid and the directly computed fine grid solutions for a NACA64A410 airfoil at $M_\infty = 0.74$ and a reduced frequency of 0.2 are shown for a number of times during an oscillation. The total oscillation starts at 1080° . It may be seen that the correction theory works fairly well except for the region just ahead of the shock. This discrepancy is due to the final interpolation in sparsely located coarse grid points and is mainly due to the coarse grid points not coinciding with the shock capture points.

Development of a Universal Indicial Function. The main objective of the work was to develop an indicial response function for transonic flow that does not require a lengthy transonic computation. If successful, this investigation would produce a means of radically reducing the computation time needed for an aeroelastic calculation. A preliminary theory has been

developed which requires steady state results at either extremity of the indicial response and some estimate of the initial time derivatives. The technique uses the strained coordinate method (reference 4) and a reparameterization of the time variable. The existing work is described in reference 6 and a typical result is shown in figure 7.

Finally, two outstanding questions about the formal strained coordinate theory have been addressed, namely the arbitrary nature of the straining and the use of constructed variables. These questions have been answered and are the subject of two short papers, references 7 and 8.

Perturbation Theory with Vanishing Shock Waves. A major restriction of the present transonic perturbation theory is that shock waves cannot be generated or destroyed during the perturbation. A study of this problem showed that the interpolation theory can be used if three (rather than two) solutions are known. The additional result is necessary because the governing equation set changes at a critical flow. In the course of this investigation several points concerning transonic flows arose. The analysis, which is based on integral equation theory, rederived Morawetz's nonexistence proofs for shock free transonic flows and also suggested that numerical algorithms which are not nonlinear may not be mathematically correct. Results for examples when shock waves vanish have been obtained. In figure 8 an example computed using two subcritical solutions and one supercritical solution is shown. Since fairly accurate subsonic solutions can be obtained (reference 9) from an incompressible solution by the use of compressibility factors, a further simplification of the theory requires only the incompressible solution and one supercritical solution. An example is given in figure 9.

Convergence Acceleration. A consequence of the mathematical perturbation theory is that data from a coarse and a medium grid numerical solution can be used to estimate the starting solution for the fine grid. The investigation showed that considerable decreases in the computation time (75% decrease) can be obtained by this means but that this improvement only occurs under certain circumstances. However, the technique does not, in any of the cases computed, significantly increase the computational time.

An alternative investigation into convergence acceleration was to couple the basic ideas of the perturbation theory with the classic ideas of convergence acceleration. The basic premise of this idea is that slow convergence of the shock location is the cause of the failure of the classic applications. A study of some computed results indicated that this hypothesis is incorrect and the investigation was terminated.

Strong Shock Potential Theory. A strong shock potential theory was derived by adding the effect of entropy production to the isentropic gas law. This theory is approximate and assumes that grid lines in the near streamwise direction are aligned with streamlines. It is also assumed that the flow is at most weakly rotational. An example is given in figure 10. It was also determined that a back-out formula does not exist. During the course of this study it was found that the conventional transonic potential theory is inconsistent since consistency requires that momentum is conserved through a shock wave. In transonic potential theory momentum is not conserved. A more consistent theory can be derived by including the momentum error in the analysis but it is physically unrealistic since the errors decrease entropy. It was also found that conventional transonic difference schemes do not conserve mass through the shock capture region although mass is conserved at the shock extremities.

Convergence Acceleration of Euler Equations. The basic idea of this study is to determine if a single equation could be constructed so as to carry the numerical errors in an Euler solution to some extent, thus avoiding the need to iterate all five conservation equations at each step. It is concluded that the modified potential equation noted in the preceding section is adequate for this task.

Truncation Error Analysis. The task was to analyze the nonlinear truncation errors of two finite difference schemes for the two-dimensional unsteady Euler equations. The method of correcting for the leading truncation error was extended for the two-dimensional problem and was very successful for the explicit scheme. The correction procedure was found to be unstable for the implicit scheme and was not extended to two dimensions. The results are given in reference 10.

Adaptive Meshes. The first research objective was to investigate means of overcoming the stiffness introduced by the adaptive meshing. The other three objectives were to determine the proper clustering functions required to minimize the truncation errors; to determine the proper smoothing and filtering operators for the truncation errors before they are suitable for use as clustering criteria; and to determine a suitable artificial dissipation to guarantee smooth and monotonic solutions at shock waves and other discontinuities.

The stiffness is a measure of the range of the eigenvalues of the flux Jacobian. The greater the range, the stiffer the problem becomes and more difficult to solve numerically. The adaptive mesh increases the stiffness in two ways; by a reduction in the smallest mesh spacing and by the mesh velocity. For explicit schemes nothing can be done about the reduced mesh spacing. However, the increased stiffness due to the mesh

velocity is more critical for explicit schemes and can be alleviated by matrix splitting. The splitting refers to the splitting of the flux Jacobian so that the effects of the convective velocities are decoupled from the effects of the mesh velocity. Since the two effects can be decoupled, each can be treated separately without loss of accuracy and hence arbitrary mesh velocities (and arbitrary mesh clustering) can be allowed in solution procedures with explicit schemes. This decoupling allows the mesh velocities to be sufficiently high so that the clustering function can keep up with the flow field features without violating the stability criteria of explicit schemes.

The most important accomplishment has been the discovery that the procedure introduced by Viviand, reference 11, for deriving the governing differential equation in the strongly conservative form of the arbitrary curvilinear coordinate system is not valid. It is commonly thought that Viviand's form of the transformed equations are the proper conservation equations which yield the correct shock strengths and speed for arbitrary mesh-clustering or mesh velocities. However, it has been shown in the present study that the shock strengths and speed are modified by the mesh clustering function and mesh velocity through the shock transition region. To obtain the proper shock jump and speed either the mesh clustering function and speed must be uniform through the shock transition region or the transition region must be of zero thickness. If the former condition is met then there is no need for the strongly conservative form of the transformed differential equation and the much simpler chain rule conservation law form (reference 12) is adequate. If the latter condition is to be met, then a shock fitting procedure is required and again the strong conservation law form is not needed.

Some numerical computations have been carried out to test the effect of mesh clustering and mesh velocity on the shock

strengths and speed and the above conclusions have been verified. These results are directly applicable to the resolution of the above mentioned research objectives. This work is reported in reference 13.

Due to the invalidity of Viviani's transformation for thick shock waves, the clustering function is no longer simply a function of the truncation errors. It must also satisfy certain restrictions so that the proper weak solution is recovered by the numerical scheme. The restriction is that the mesh should be nearly uniform throughout the shock (or contact surface) transition region. Since this is also the region where the truncation errors vary most rapidly there is no possibility of adapting the mesh so that the truncation error is uniform over the entire computational domain. The mesh induced truncation errors can be greatly reduced (actually completely eliminated) if the fine but uniform mesh occurs only in regions where the solution truncation errors are large and the coarse mesh is only in regions where the truncation errors are small. The transition between the fine and coarse mesh need not be smooth provided that they occur only in the regions where the solution is locally uniform and the mesh transformation metrics are computed according to reference 12.

The third research objective was to obtain the proper filtering and smoothing the functions for the truncation errors. The truncation errors can be considered to be wave packets moving along with the features of the flow field. The purpose of filtering and smoothing the truncation error is to find the envelope of the packet. The details within the wave packets are not important. The nonlinear truncation error analysis provides not only the envelope but also the details within the wave packet. The envelope can be determined in many cases by the curvature of the numerical solution. Thus, it is more efficient in most cases

simply to look at the curvature of the numerical solution as an approximation of the lowest order harmonics of the truncation errors. The exact nonlinear truncation errors are not required. This is fortunate since it is quite expensive to compute the nonlinear truncation errors.

The final research objective was to investigate the mesh dependent dissipation required to obtain monotonic and smooth shock waves. It is known (reference 14) that to obtain monotonic solutions at shock waves that the mesh spacing must not exceed a critical value set by the local amount of dissipation (either artificial or numerical). If, however, the mesh spacing is very much less than the critical value then the shock becomes excessively diffused or smeared resulting in a loss of the effective use of the available number of mesh points. To utilize this trick in an adaptive mesh strategy it is necessary to know the numerical dissipation rate, which is difficult to determine since it is a nonlinear function of both the solution and the metrics. The mesh dependent artificial dissipation was not too successful in producing monotonic and smooth shock waves.

The above work was extended to the two-dimensional unsteady Euler equation for both explicit and implicit second order accurate schemes. This was done by rederiving the nonlinear modified equations for the Euler equations in an arbitrarily moving coordinate system. This truncation error analysis provided for a rational basis of the solution adaptive mesh. Several mesh generators were investigated and the ones based on variational principles were found to be the most effective. The mesh generator is rather costly in terms of computational effort. A weighted interpolation procedure was developed so that only a coarse mesh needed to be computed by the mesh generator. The final mesh is then obtained by the weighted interpolation procedure.

Transonic Multiple Solutions. In recent years multiple solutions to the numerical approximation to the full potential equations have appeared in the literature (references 15 and 16). Initially the phenomena appeared in computations of the flow over a symmetric airfoil at zero angle of attack when two lifting solutions were present in addition to the expected non-lifting solution. In reference 16 some results for a non-symmetric airfoil, a RAE 2822 section, are also presented. Steinhoff and Jameson (reference 16) suggested that the change from one of the solutions to another is discontinuous and noted a hysteresis effect indicating that the lift coefficient (C_L) depended on whether the angle of attack (α) was increasing or decreasing. More recent work is by Salas (reference 17) who has extended the computations of the flows considered by Steinhoff and Jameson (references 15 and 16) to show that it is possible to construct a smooth $C_L - \alpha$ curve connecting the three solutions for a symmetric airfoil.

The investigations noted above are meticulously performed and are essentially numerical experiments. There is a limited amount of understanding that can be gained from such experiments and consequently a more analytic technique may yield more information. Furthermore, although the numerical results are invaluable they do not exclude the possibility that the multiple solutions are due to the numerical approximation to the differential equation. The present investigation is based on the integral equation formulation (reference 18) which allows some insight into the problem.

The transonic integral equation method of reference 18 is only applicable to the transonic small disturbance (TSD) equation rather than the full potential equation (FPE) that is used in the earlier work. Consequently, the first step was to reproduce multiple solutions using the TSD equation. Once these solutions

were obtained they were analyzed using the ideas of the transonic integral equation theory. In this investigation these suggestions have been implemented and the conclusions are as follows. The study indicated that the formulation of the TSD equation (and by implication the FPE) is not unique even with the Kutta condition enforced. The formulation indicated that eigen-solutions can exist which can be combined with the correct solution to give erroneous results. These eigensolutions introduced arbitrary constants into the solution and a preliminary examination indicated that there is no obvious means of determining these constants. This work is reported in reference 19.

PERSONNEL

The Principal Investigator on the steady and unsteady perturbation theory is Dr. David Nixon. Dr. Samuel C. McIntosh is the Co-Principal Investigator on the unsteady flow aspects of the problem and Dr. G. David Kerlick is closely associated with the work. For the numerical analysis aspects Dr. Goetz H. Klopfer is Co-Principal Investigator.

The Principal Investigator on the transonic theory is Dr. David Nixon. For the adaptive grid aspects, Dr. Goetz H. Klopfer is Co-Principal Investigator. Dr. David S. McRae of North Carolina State University is a subcontractor of this work.

PAPERS AND REPORTS

1. Nixon, D.: Development of Perturbation Procedures for Non-linear Inviscid and Viscous Flows. AFOSR-TR-80-0129, 1980.
2. Nixon, D.: Perturbation Methods in Transonic Flow. AIAA Paper 80-1367, 1980.
3. Nixon, D.: Observations on the Strained Coordinate Method for Transonic Flows. AIAA Journal, Vol. 18, No. 3, 1980.

4. Nixon, D.: Transonic Small Disturbance Theory with Strong Shock Waves. AIAA Journal, Vol. 18, No. 6, 1980.
5. Kerlick, G. D. and Nixon, D.: Calculation of Unsteady Transonic Flows by the Indicial Method. Journal of Applied Mechanics, Vol. 49, No. 2, 1982.
6. Nixon, D. and McIntosh, S. C., Jr.: Further Observations on the Strained Coordinate Method for Transonic Flows. AIAA Journal, Vol. 18, No. 12, 1980.
7. McIntosh, S. C. and Nixon, D.: On the Use of Constructed Variables in the Method of Strained Coordinates. AIAA Journal, Vol. 19, No. 3, 1981.
8. Nixon, D.: On the Derivation of Universal Indicial Function. AIAA Paper 81-0328, 1981.
9. Klopfer, G. H. and McRae, D. S.: The Nonlinear Modified Equation Approach to Analyzing Finite Difference Schemes. AIAA Paper 81-1029, 1981.
10. Nixon, D.: Some Comments on the Correction Theory for Transonic Viscous Flow. NEAR Paper 152, 1982.
11. Kerlick, G. D. and Nixon, D.: Convergence Acceleration for Steady Transonic Flow Calculations by Grid Refinement Correction. NEAR Paper 157, 1982.
12. Nixon, D. and Klopfer, G. H.: Some Remarks on Transonic Potential Flows. Journal of Applied Mechanics, Vol. 50, No. 2, 1983.
13. Nixon, D.: Perturbation Procedures for Nonlinear Viscous Flows. Journal of Applied Mechanics, Vol. 50, No. 2, 1983.
14. Klopfer, G. H. and Nixon, D.: Nonisentropic Potential Formulation for Transonic Flows. AIAA Paper No. 83-0375, 1983, (in press AIAA Journal, 1984).
15. Nixon, D. and Kerlick, G. D.: Perturbation of Transonic Flow with Vanishing Shock Waves. (to be published in AIAA Journal, 1984)
16. Klopfer, G. H.: Weak Solutions, Mesh Transformation, and Forms of Conservation Law Equations. NEAR Paper 162, May 1982. (to be submitted to AIAA Journal)

17. Nixon, D.: Behavior of the Flow Through a Numerically Captured Shock Wave. AIAA Journal, Vol. 22, No. 1, 1984.
18. Nixon, D.: Observations on the Occurrence of Multiple Solutions in Transonic Flow. NEAR Paper No. 179, February 1983. (submitted to Journal of Applied Mechanics)
19. Nixon, D.: The Grid Correction Theory In Unsteady Flows. NEAR Paper No. 180, 1983.
20. Klopfer, G. H.: Solution Adaptive Meshes for the 1-D Euler Equations - Explicit Schemes. NEAR Paper No. 181, February 1983.
21. Klopfer, G. H.: Solution Adaptive Meshes for the Two-Dimensional Unsteady Euler Equations - Explicit Schemes. NEAR Paper No. 189, 1984. (to be submitted to AIAA Journal)
22. McRae, D. S.: Solution Adaptive Meshes for the Unsteady Euler Equations - Implicit Schemes. North Carolina State University Engineering Report, April 1984. (to be submitted to AIAA Journal)

PRESENTATIONS

1. Nixon, D.: Calculation of Unsteady Transonic Flows by the Indicial Method. Workshop on Unsteady Transonic Aerodynamics, NASA/Langley Research Center, February 1980.
2. Nixon, D.: Perturbation Methods in Transonic Flow. Seminar Series in Aeronautics and Astronautics, Stanford University, March 1980.
3. Nixon, D.: Perturbation Methods in Transonic Flow. AIAA Fluid & Plasma Dynamics Conference, Snowmass, Colorado, July 1980.
4. Nixon, D.: On the Derivation of Universal Indicial Functions. AIAA Aerospace Sciences Conference, St. Louis, Missouri, January 1981.
5. McIntosh, S. C., Jr.: Rapid Estimation of Aerodynamic Forces by the Transonic Indicial Method. University of California, Los Angeles, California, School of Engineering, May 1981.

6. Klopfer, G. H.: The Nonlinear Modified Equation Approach to Analyzing Finite Difference Schemes. AIAA 5th Computational Fluid Dynamics Conference, Palo Alto, June 1981.
7. Nixon, D.: Some Fundamental Aspects of Transonic Flow Theory. Stanford University, November 1982.
8. Nixon, D.: Some Fundamental Aspects of Transonic Flow Theory. NASA/Langley Research Center, December 1982.
9. Klopfer, G. H.: Non-Isentropic Potential Formulation for Strong Shocks. NASA/Ames Research Center, January 1983.
10. Klopfer, G. H.: Non-Isentropic Potential Formulation for Strong Shocks. AIAA Aerospace Sciences Conference, Reno, Nevada, January 1983.
11. Nixon, D.: Multiple Solutions in Transonic Flow. NASA/Ames Research Center, February 1983.
12. Nixon, D.: Multiple Solutions in Transonic Flow. NASA/Ames Research Center, April 1983.
13. Nixon, D.: Multiple Solutions in Transonic Flow. NASA/Langley Research Center, April 1983.
14. Nixon, D.: Multiple Solutions in Transonic Flow. NASA/Lewis Research Center, April 1983.
15. Nixon, D.: Some Recent Topics in Transonic Flow. United Technologies Research Center, October 1983.

TECHNICAL APPLICATIONS

The most recent application of the research developed under this contract was the simulation of aileron buzz on an experimental wing section designed by Gates-Learjet. The indicial response was generated by the Computational Fluid Dynamics Branch at NASA/Ames Research Center who solved the Reynolds averaged Navier-Stokes equations. The aerodynamic response was directly coupled to a structural model of the aileron. The indicial method gives enormous savings in computer time since a single direct calculation takes about two hours on the ILLIAC IV computer.

Most of the research sponsored under the present contract is fundamental and the development of applications is proceeding under alternate sponsorship. Developments of this research have been sponsored by the following organizations:

NASA/Ames Research Center (Applied Computational Aerodynamics Branch)

NASA/Ames Research Center (Computational Fluid Dynamics Branch)

Naval Air System Command

Lockheed-Georgia Company

Office of Naval Research

Applied Technology Laboratory, USARTL (AVRADCOM)

The computer code developed to treat strong shock waves in potential theory has been requested by NASA/Ames Research Center to use with a boundary layer code.

REFERENCES

1. Nixon, D.: Perturbation of a Discontinuous Transonic Flow. AIAA Journal, Vol. 16, No. 1, 1978.
2. Nixon, D.: Perturbations in Two and Three Dimensional Transonic Flow. AIAA Journal, Vol. 16, No. 7, 1978.
3. Nixon, D.: Transonic Small Disturbance Theory with Strong Shock Waves. AIAA Journal, Vol. 18, No. 6, 1980.
4. Nixon, D.: Notes on the Transonic Indicial Method, AIAA Journal, Vol. 16, No. 6, 1978.
5. Nixon, D.: Techniques for Correcting Approximate Finite Difference Solutions. AIAA Paper 79-0277, 1979.
6. Nixon, D.: On the Derivation of Universal Indicial Functions. AIAA Paper 81-0328, 1981.

7. Nixon, D. and McIntosh, S. C., Jr.: Further Observations on the Strained Co-Ordinate Method for Transonic Flows. AIAA Journal, Vol. 18, No. 12, 1980.
8. McIntosh, S. C., Jr. and Nixon, D.: On the Use of Constructed Variables in the Method of Strained Co-Ordinates AIAA Journal, Vol. 19, No. 3, 1981.
9. Nixon, D. and Hancock, G. J.: Integral Equation Methods - A Reappraisal. Symposium Transonicum. Springer-Verlag, 1976.
10. Klopfer, G. H. and McRae, D. S.: The Nonlinear Modified Equation Approach to Analyzing Finite Difference Schemes. AIAA Paper 81-1029, 1981.
11. Viviand, H.: Conservation Forms of Gas Dynamic Equations. La Recherche Aerospatiale, No. 1, January-February 1974, pp. 65-68.
12. Hindman, R. D.: Geometrically Induced Errors and Their Relationship to the Form of the Governing Equations and the Treatment of Generalized Mappings. AIAA Journal, Vol. 20, No. 10, 1982.
13. Klopfer, G. H.: Weak Solutions, Mesh Transformation, and Forms of Conservation Law Equations. NEAR Paper 162, May 1982 (to be submitted to AIAA Journal)
14. Miller, K. and Miller, R. N.: Moving Finite Elements - I. SIAM Journal of Numerical Analysis, Vol. 18, 1981.
15. Steinhoff, J. S. and Jameson, A.: Nonuniqueness in Transonic Potential Flows in Numerical Methods for Computations of Inviscid Transonic Flows with Shock Waves. Editors, A. Rizzi, H. Viviand, Vieweg Braunschweig, 1981.
16. Steinhoff, J. S. and Jameson, A.: Multiple Solutions of the Transonic Potential Flow Equation. AIAA Journal, Vol. 20, No. 11, 1982.
17. Salas, M. D., Gumbert, C. R., and Turkel, E.: Nonunique Solutions to the Transonic Potential Flow Equation. AIAA Journal, Vol. 22, No. 1, 1984.
18. Nixon, D.: Calculation of Transonic Flows Using an Extended Integral Equation Method. AIAA Journal, Vol. 15, No. 3, 1977.

19. Nixon, D.: Observations on the Occurrence of Multiple Solutions in Transonic Flow. NEAR Paper No. 179, February 1983. (submitted to Journal of Applied Mechanics)

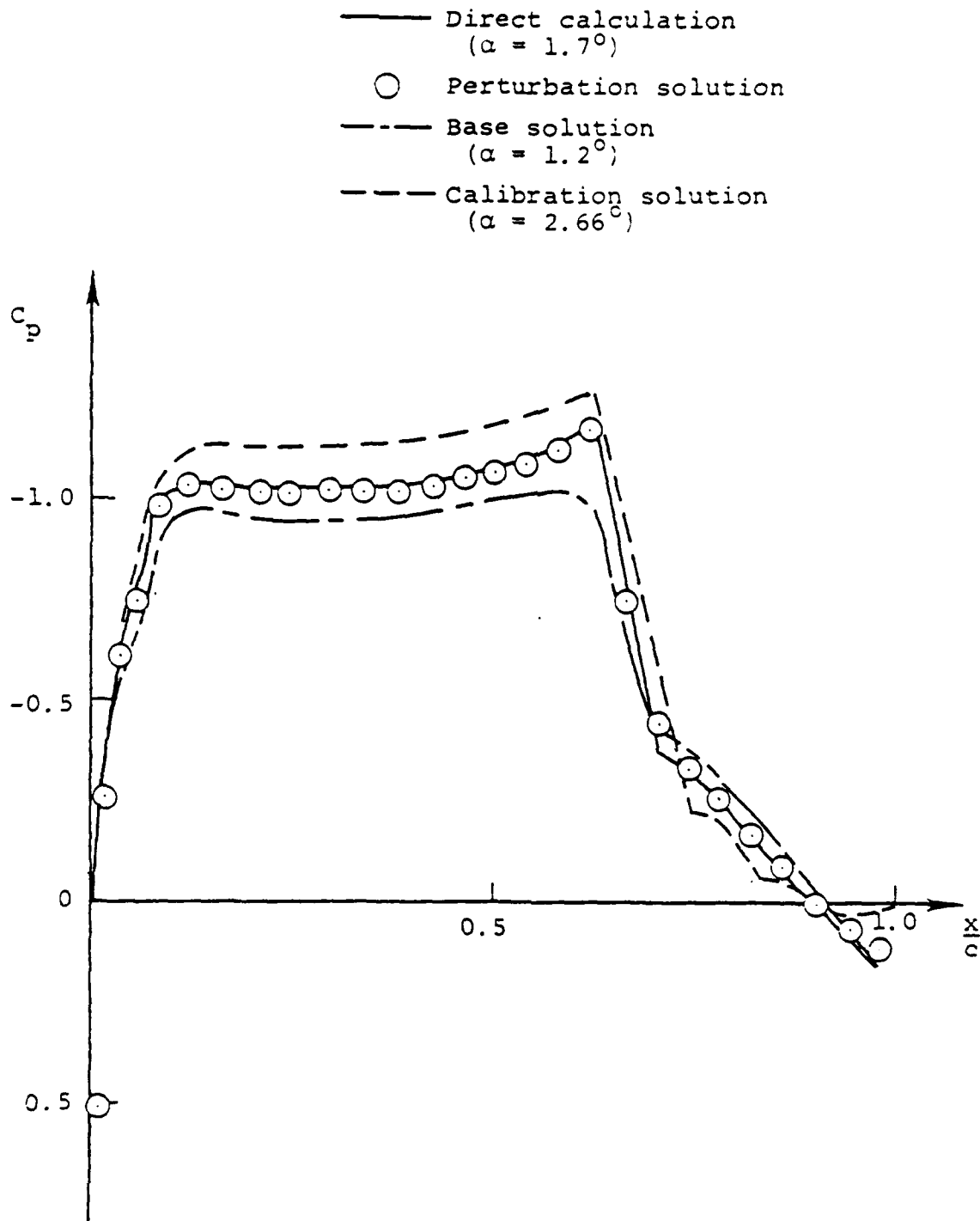


Figure 1.- Pressure distribution around the upper surface of Korn airfoil; $M_\infty = 0.755$, $\alpha = 1.7^\circ$.

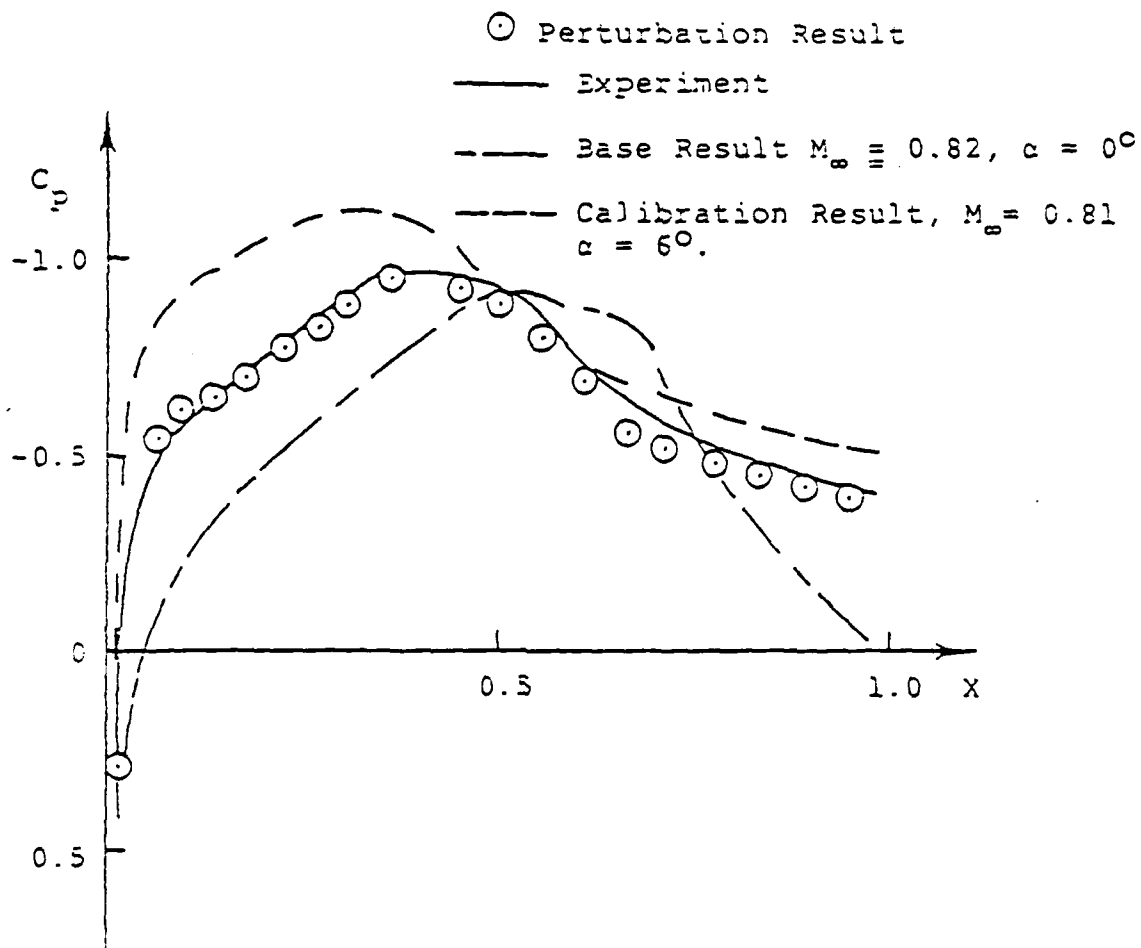


Figure 2.- Pressure distribution on the upper surface of a NACA 64A410 Airfoil; $M_\infty = 0.82, \alpha = 4^\circ$.

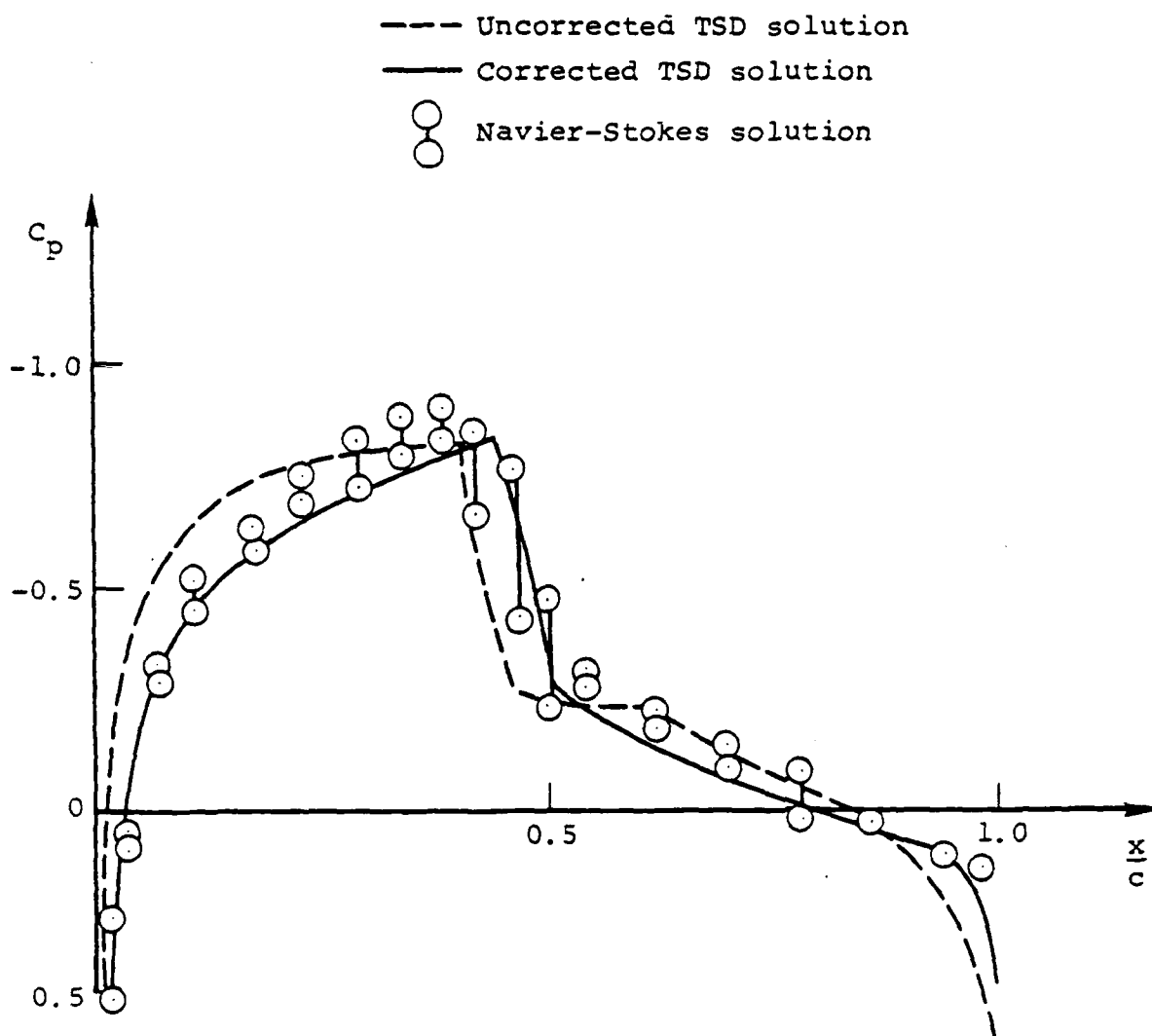
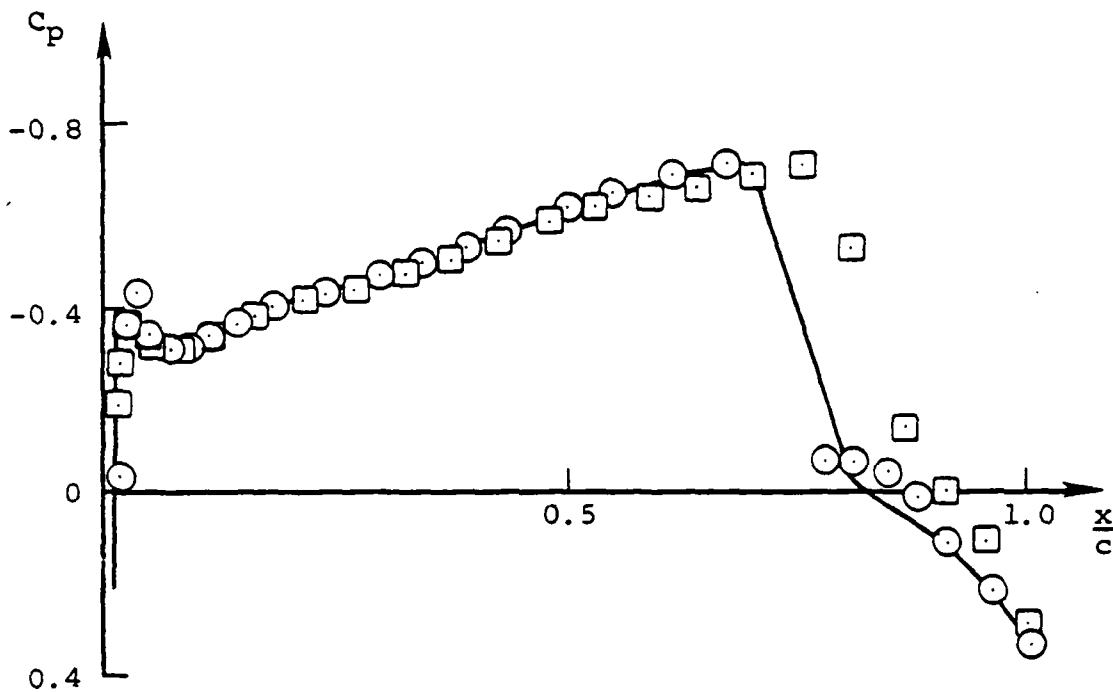


Figure 3 .- Pressure distribution around a NACA0012 airfoil; $M_\infty = 0.8$, $\alpha = 0^\circ$.

$$\eta = 0.0$$

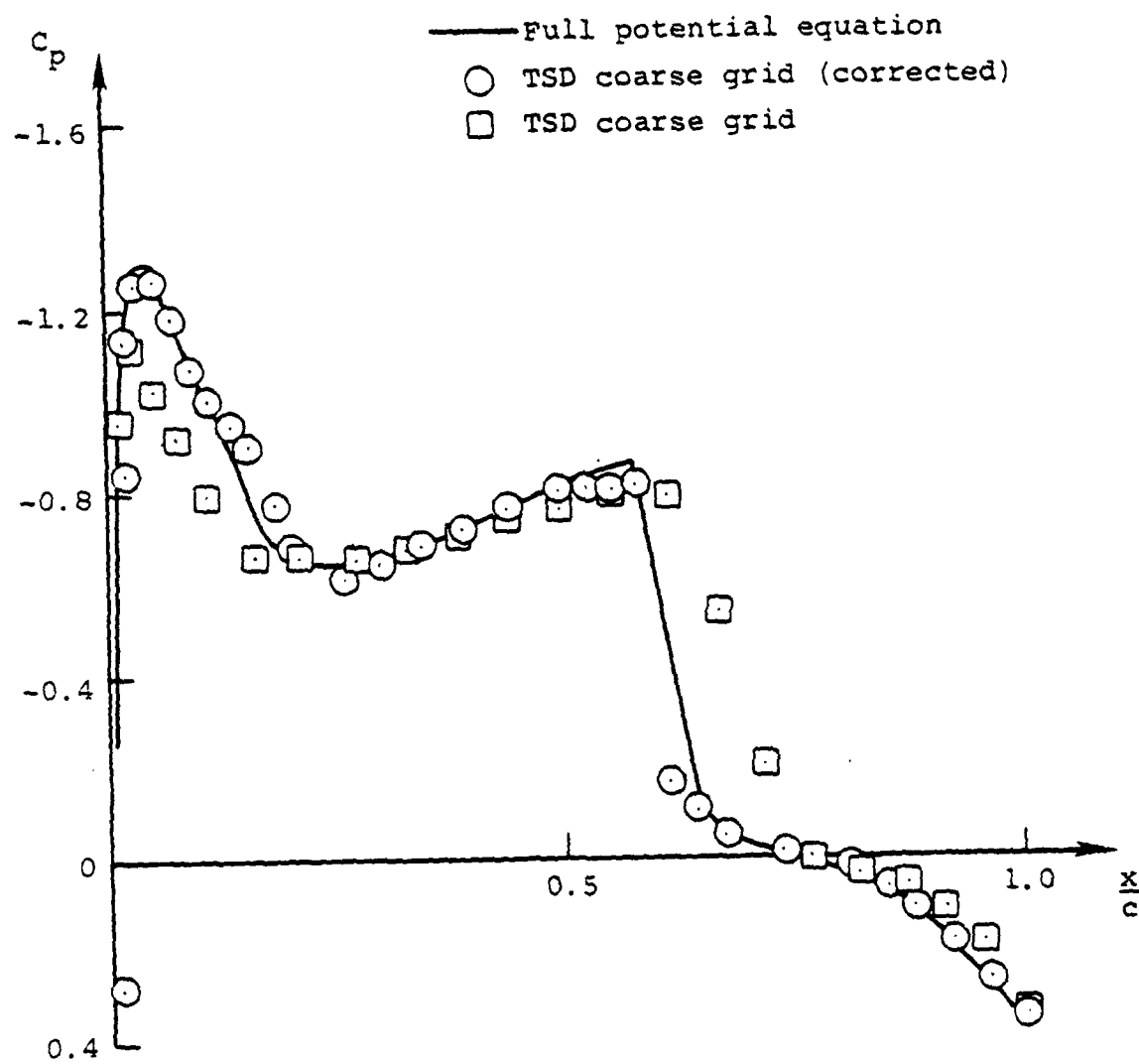
- Full potential equation
- TSD coarse grid (corrected)
- TSD coarse grid



(a)

Figure 4.- Pressure distribution over the upper surface of the ONERA M6 wing; $M_\infty = 0.84$, $\alpha = 4^\circ$, $\theta_{\text{twist}} = 1.5^\circ$ (TSD coarse grid/full potential equation correction).

$$\eta = 0.35$$



(b)

Figure 4 .- Continued.

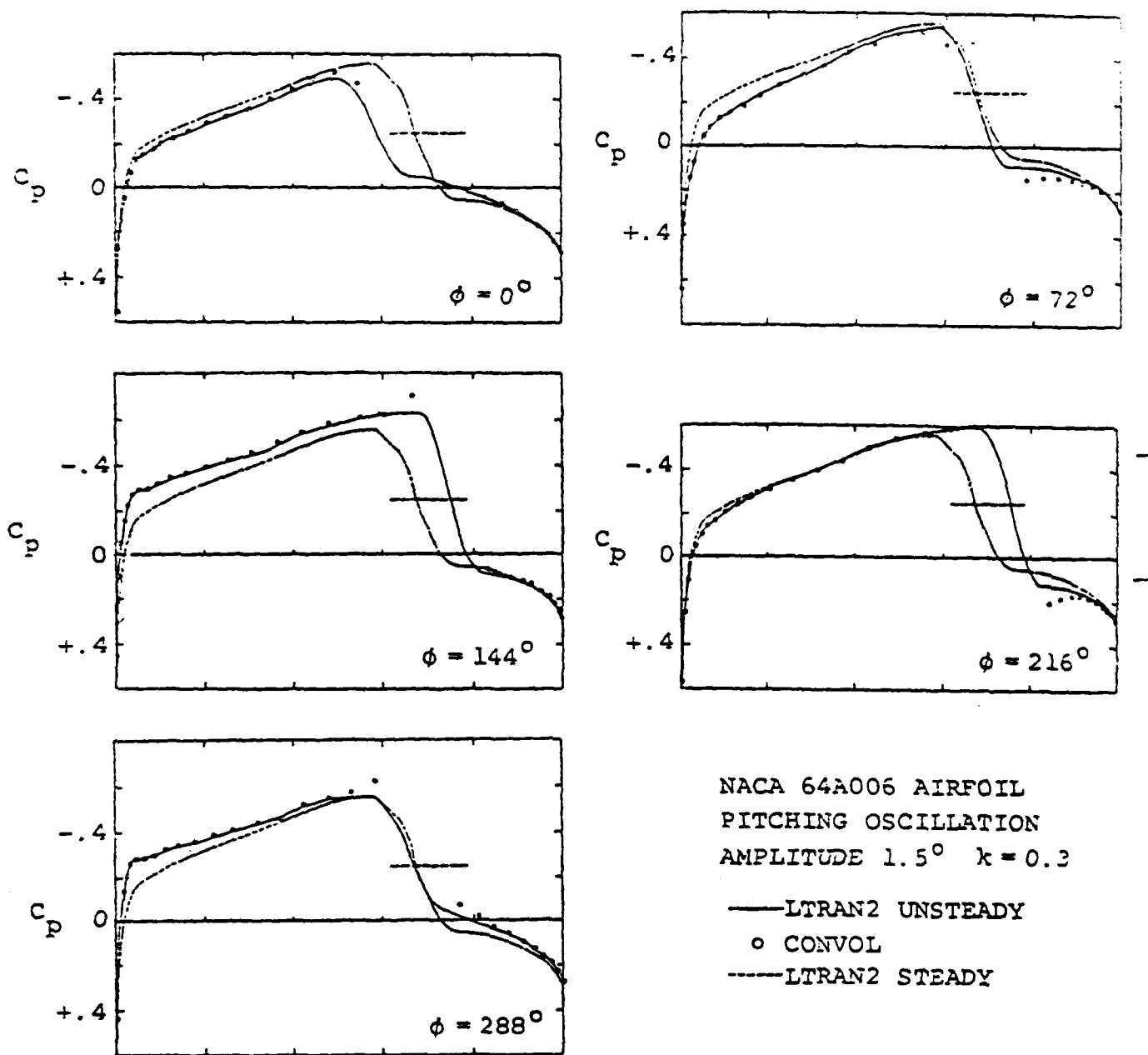


Figure 5.- Unsteady pressure distributions around an airfoil oscillating in pitch.

NACA64A410 IT. 750 T= 1080. DEG

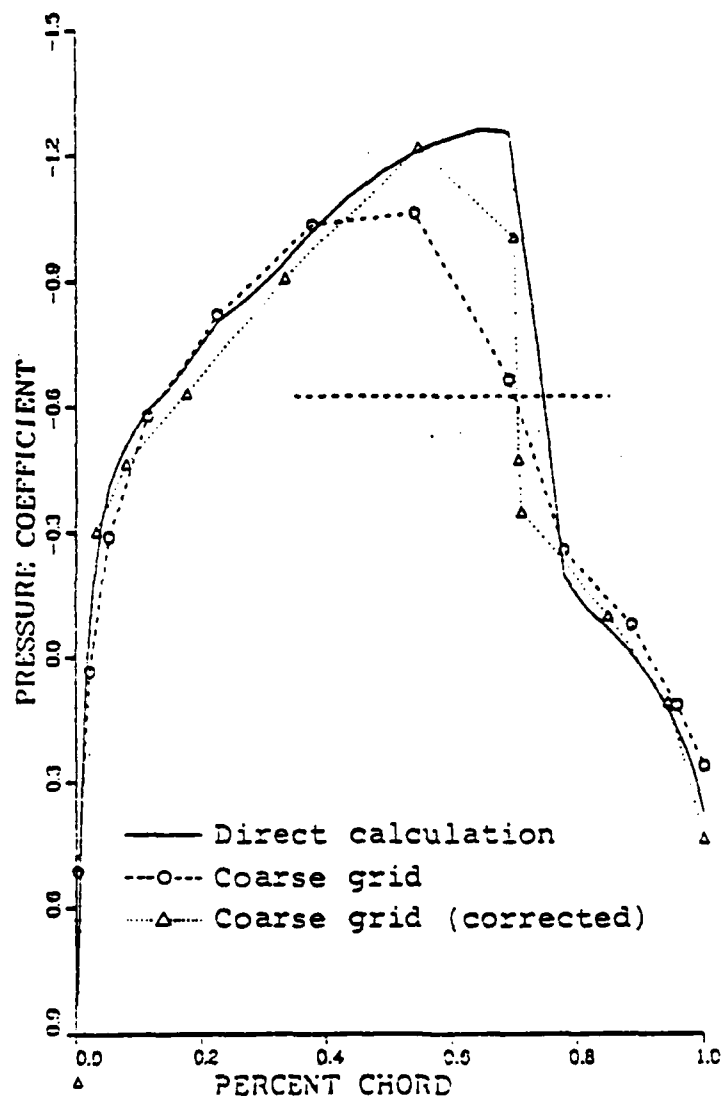


Figure 6.- Application of the correction theory to unsteady flows.

NACA64A410 IT. 850 T= 1224. DEG

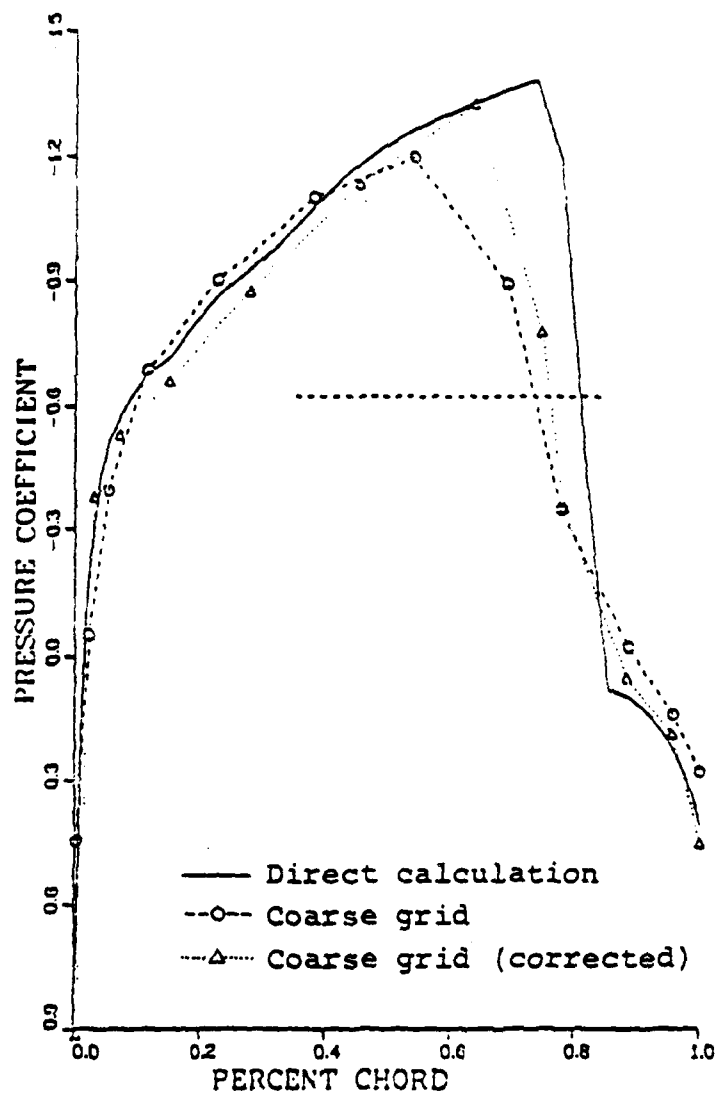


Figure 6.- Concluded.

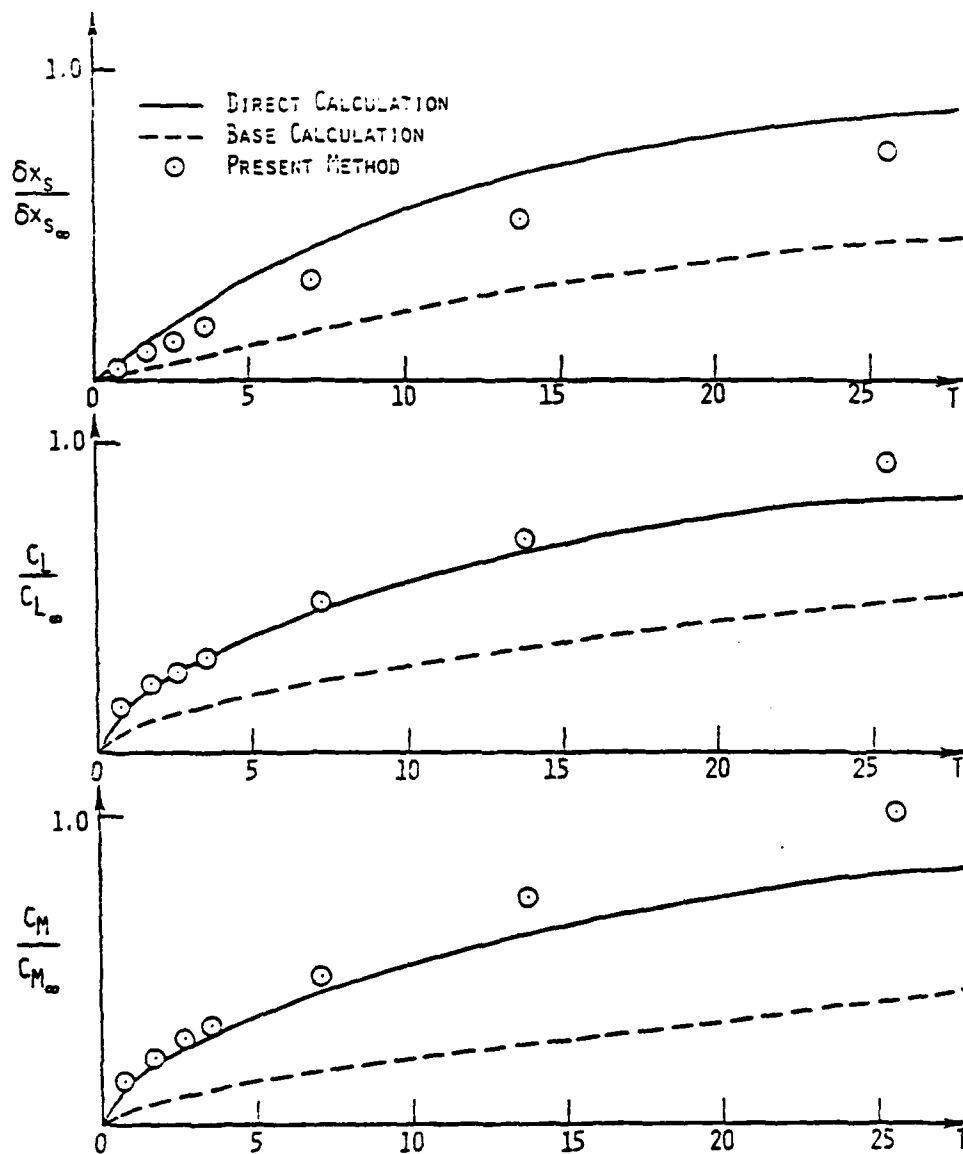


Figure 7. - Variation of $\delta x_s/\delta x_{s_\infty}$, C_L/C_{L_∞} , C_M/C_{M_∞} with time for a NACA 0012 airfoil; $M_\infty = 0.8$, $k = 0.7$, $\bar{k} = 0.8$.

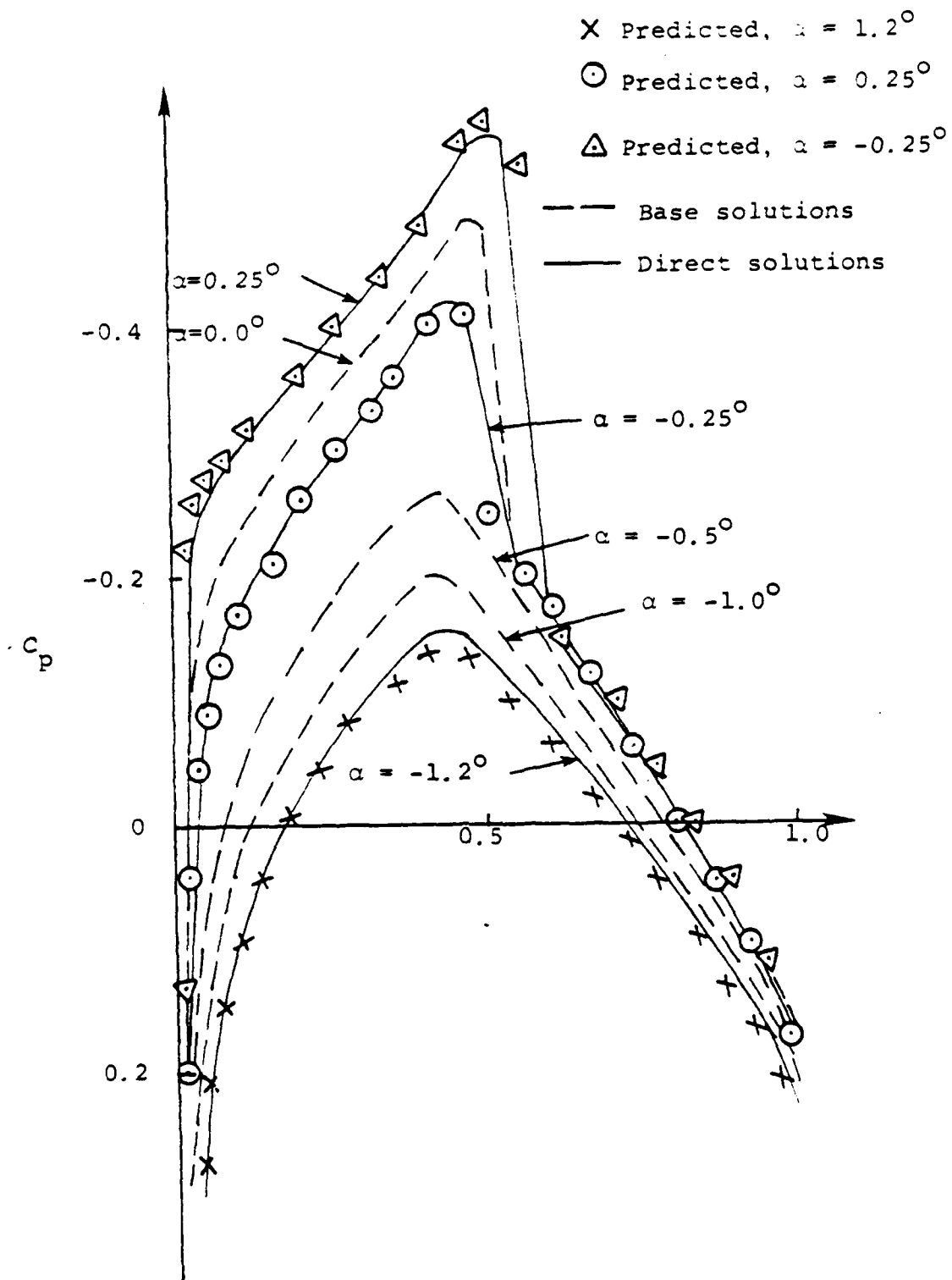


Figure 8.- Interpolation theory for the upper surface
 an NACA 64A006 airfoil at $M_\infty = 0.85$.

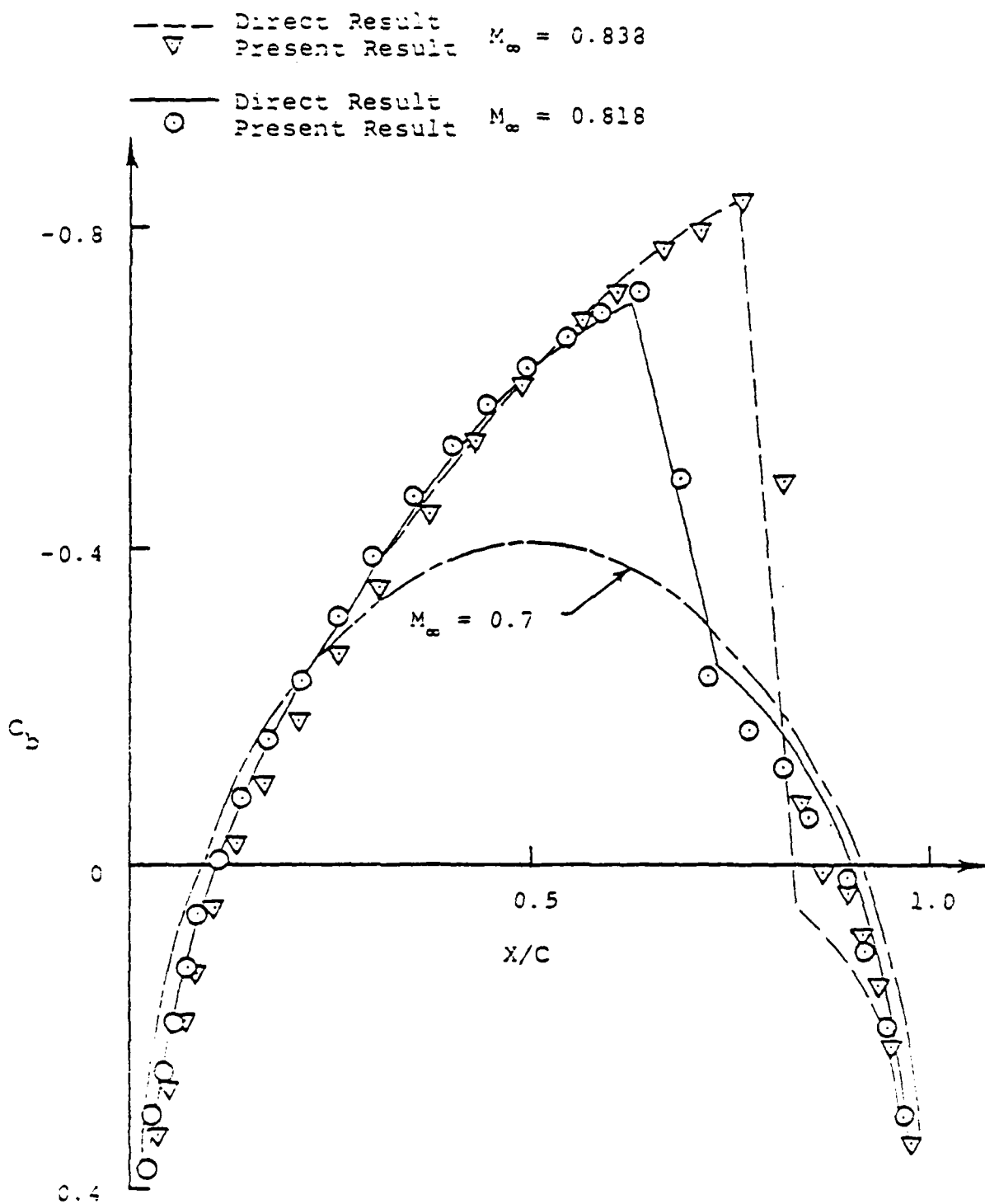


Figure 9.- Pressure distribution around a 10% Biconvex Airfoil.

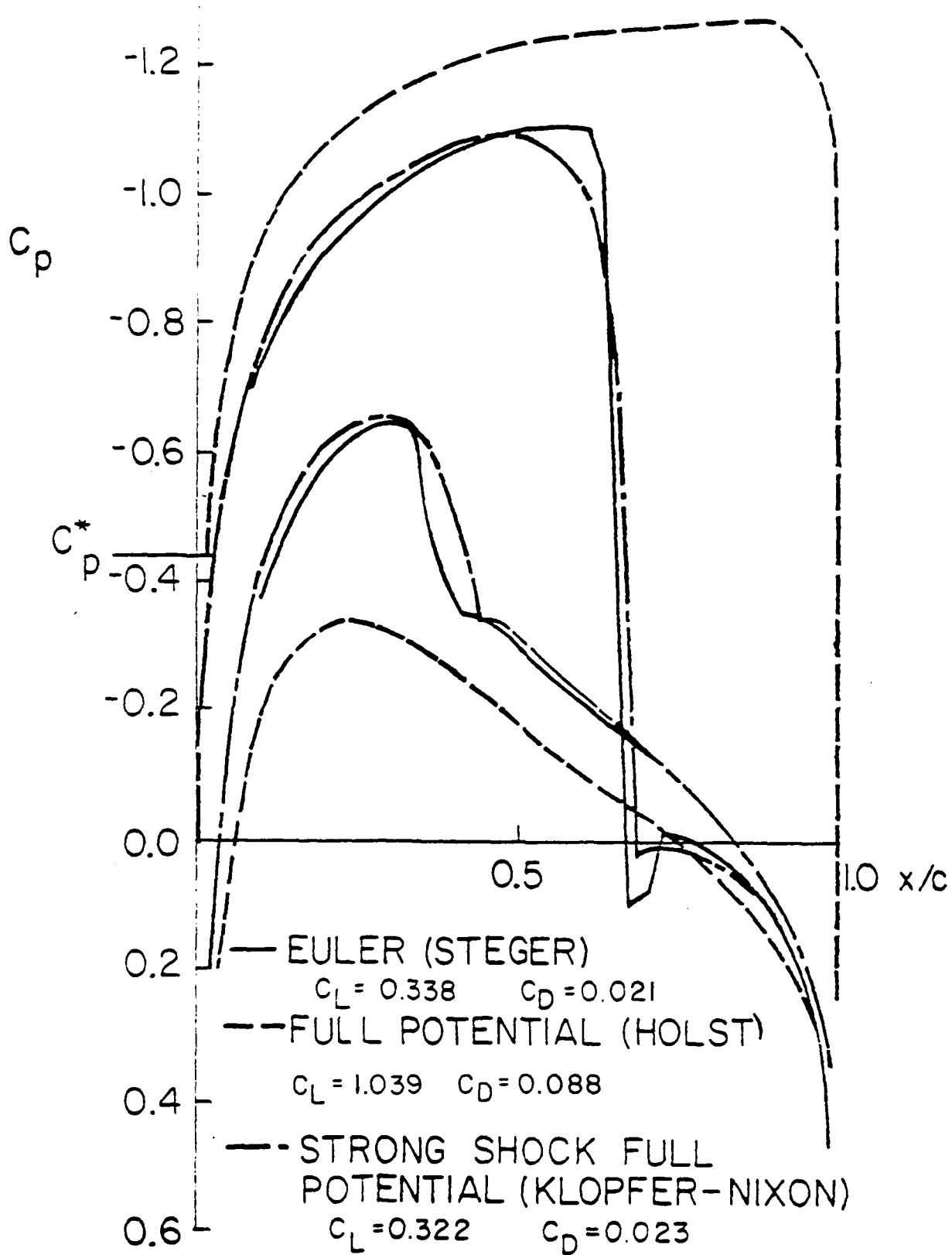


Figure 10.- Pressure distribution around a NACA 0012 airfoil at $M_\infty = 0.80$, $\alpha = 1.25^\circ$.

END

FILMED

84

111